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ANALYZING THE INFLUENCE OF SCREENING PARAMETERS ON NEUTRINO INTERACTIONS

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Abstract

This article discusses the nature of exchange potential between neutrinos and particles. The study shows the variation of exchange potential between three types of neutrinos (muon, electron, and tau) and different types of particles (proton, neutron, and a mixture of neutron and proton). The exchange potential is found to be higher for lower order Bessel functions and lower for higher order Bessel functions, and the potential remains the same for all cases with very little variation in potential with different screening parameters and neutrino masses. The results of the study are consistent with previous research on the effect of screening parameters and neutrino mass on the exchange potential. However, the present study provides a more comprehensive understanding of the exchange potential by investigating the effect of different types of particles, screening parameters, and Bessel functions. Previous studies have explored the exchange potential between neutrinos and nucleons and found that the exchange potential increases with increasing screening parameters and decreasing neutrino mass, which is consistent with the findings of the present study. Another study found that the exchange potential is affected by the isospin states of the nucleons and can be used to distinguish between them. Overall, the present study contributes to a better understanding of the exchange potential between neutrinos and particles.

Keywords: Bessel Function, Exchange Potential, Neutrino Masses, Screening Parameters.

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1. Introduction

Neutrino oscillations, a phenomenon that revealed features beyond the Standard Model, is a major accomplishment in particle physics. Despite progress, complete knowledge of the many elements of neutrino oscillations remains a long way off. Unsolved challenges and prospects for future resolution are explored. Neutrino research has led to significant advances, but the properties of neutrinos remain poorly understood. The discovery of oscillation phenomena threw the Standard Model into disarray, providing proof of nonzero neutrino mass. The hypothesis of neutrino oscillations was proposed before any scientific evidence in 1957 by Pontecorvo. Davis, following a method suggested by Pontecorvo in 1948, made the first effort to detect the final states created by neutrino interactions in 1955. Reines and Cowan used a reaction outlined by Bethe and Peierls in 1934 to make the first successful measurement in 1956.

In the 1940s and 1950s, Pontecorvo and Puppi proposed the existence of new neutrinos corresponding to the muon and tau particles. Lederman, Schwartz, and Steinberger demonstrated the difference between the muon and electron neutrinos in 1962. Despite decades of research, several questions about neutrino oscillations remain unanswered, such as oscillation parameters, neutrino masses, mass hierarchy, CP violation, and the possibility of a fourth, sterile neutrino. Hypotheses beyond the Standard Model propose nonstandard neutrino interactions and the presence of a light sterile neutrino that could modify this transition. Neutrinos provide undistorted information about their sources, allowing researchers to learn about the character of neutrinos and the interiors of celestial objects [1].

The idea of neutrino oscillations, which involves the mixing of different neutrino families, was introduced in the 1960s by several researchers. Wolfenstein discovered the matter effect in 1978, which explains how neutrinos propagate in ordinary matter. Evidence of neutrino oscillations has been gathered from monitoring solar and atmospheric neutrinos, and the Sudbury Neutrino Observatory and Super-Kamiokande experiments provided key evidence in 2015. These discoveries have led to a better understanding of neutrino masses, the number of neutrino families, and their interactions with matter [2].

The authors of the paper investigate how the long-range effective potential, resulting from the exchange of a pair of massless neutrinos between two objects separated by r, might change if r is small enough to invalidate the contact interactions. They investigate two scenarios involving a t-channel or an s-channel mediator to open up touch interactions, and construct a basic formula that may be used to define the potential in all regimes. The potential drops as 1/r5 near the long-range limit in both scenarios, but the s-channel potential exhibits 1/r4 and 1/r2 behaviors in the short-range limit [3]. There is no compelling direct evidence for large neutrinos to date, and the current top bounds for ν_e, ν_μ, and ν_τ [4].

$$m(\nu_e) \leq 7eV, m(\nu_\mu) \leq 0.27MeV, m(\nu_\tau) \leq 31eV \qquad (1)$$

$G_F = 1.16639 \times 10^{-5}\, GeV^{-2}\, (\hbar c)^3$ is the Fermi decay constant. Subsequently Feinberg, Sucher, and Au (FSA) recalculated $V_{ee}^{(2)}(r)$ in the framework of the Standard Model and obtained

$$V_{ee}^{(2)}(r) = G_F^2 \left(2\sin^2\theta_W + \frac{1}{2}\right)^2 \left(\frac{1}{4\pi^3 r^5}\right) \qquad (2)$$

where θ_W is the weak mixing angle, with $\sin^2\theta_W = 0.2319(5)$. The result in eq. (2) has been rederived recently by Hsu and Sikivie using a different formalism from that of FS. The magentiude of corresponding force $\vec{F}(r) = -\vec{\nabla} V_{ee}^{(2)}(r)$ [5].

When a W boson, a neutrino, and a charged lepton interact, they can cause events such as a charged lepton turning into a neutrino and emitting a (virtual) W boson, or a W boson creating a pair of neutrinos and charged leptons. In the decay of a (virtual) W^- or W^+, one would write as shown in [4].

$$W^+ \rightarrow \ell^+ + \nu_\ell, W^- \rightarrow \ell^- + \bar{\nu}_\ell \qquad (3)$$

In beta decay, a heavy nucleus (A, Z) undergoes radioactivity and transforms into a lighter nucleus with the same mass number but a different charge by emitting an electron (or positron). The label $l = e, \mu, \tau$, used for the neutrino represents the type of charged lepton it corresponds to.

$$(A, Z) \rightarrow (A, Z \pm 1) + e^{\pm} + \text{nothing else visible} \qquad (4)$$

The decay of pions and muons with muon neutrinos and anti-neutrinos:

$$\pi^+ \rightarrow \mu^+ + v_\mu, \qquad \mu^- \rightarrow e^+ + v_e + \bar{v}_\mu \tag{5}$$

After the discovery of the decay $\pi^{\pm} \rightarrow \mu^{\pm} + \nu$, scientists wondered if the undetected particles produced in the decay were the same as the neutrinos produced in β decay. The interaction between active neutrinos is still not well understood due to the elusive nature of neutrinos. Lab experiments have not yet measured neutrino-neutrino scattering. While the coupling of neutrinos with the Z-boson has been measured, the possibility of stronger self-interaction of neutrinos through light mediators cannot be excluded. A lepton number charged scalar or Majoron, which is a SM gauge singlet that exclusively couples with neutrinos at the tree level, can provide a viable window for this interaction [6-8].

The upper limit of the effective electron anti-neutrino mass (m_ν) determined from the Karlsruhe Tritium Neutrino experiment's second physics run. The experiment measures the tritium β-decay spectrum close to its endpoint, which allows for a precise determination of m_ν without relying on any cosmological model or assuming whether the neutrino is a Dirac or Majorana particle. By increasing the source activity and reducing background, the experiment reached a sensitivity of 0.7 eVc^{-2} at a 90% confidence level. The study's best-fit spectral data yields $m_\nu^2 = (0.26 \pm 0.34) eV^2c^{-4}$, which gives an upper limit of m_ν [9].

The interaction of neutrinos with other particles is crucial to our understanding of fundamental physics, astrophysics, and cosmology. However, the nature and properties of the neutrino exchanging potential are not well understood. The goal of this research is to investigate the neutrino exchanging potential and its underlying mechanisms. Specifically, this study aims to identify any new insights into the behavior of neutrinos and to contribute to the development of more accurate models and predictions in the field of neutrino physics. This problem statement outlines the significance of the research question and the specific objectives of the study, which will guide the research methodology and data analysis.

Neutrinos are elementary particles that have very weak interactions with matter. Understanding their interactions with other particles, including the neutrino exchanging potential, is a relatively new field of study that requires advanced experimental and theoretical techniques. Neutrinos play a crucial role in many astrophysical phenomena, such as supernovae and the cosmic microwave background radiation. They are also key components of the Standard Model of particle physics and can help us understand the fundamental nature of matter and energy. Therefore, studying the neutrino exchanging potential is crucial for advancing our understanding of these phenomena. Neutrinos are notoriously difficult to detect and study due to their weak interactions. Studying the neutrino exchanging potential is necessary for accurately modeling and predicting neutrino interactions in various physical scenarios. This can have practical applications, such as improving the design and performance of neutrino detectors and contributing to the development of new technologies in fields such as energy and medicine.Methodology

The dependence on transferred momentum and modification of nucleon dispersion relations on mean field level are present in leptonic and semi-leptonic processes. The flavor-exchange reactions of $\nu_e + \mu^- \rightarrow \nu_\mu + e^-$ and $\bar{\nu}_\mu + \mu^- \rightarrow \bar{\nu}_e + e^-$ are dominant at low energies, while the capture of ν_e on neutron and scatterings of $\bar{\nu}_\mu$ on nucleons are sources of opacity for these species. The inverse muon decay $\bar{\nu}_e + \nu_\mu + e^- \leftrightarrow \mu^-$ can also dominate over the scatterings of $\bar{\nu}_e$ and ν_μ on nucleons at low energies. At high energies, the corrections in semi-leptonic processes become more significant [10]. The flavors of neutrinos is the partner of a charged lepton (electron, muon, tau), connected to it by the weak interaction as :

$$\mathcal{L} = \sum_{\alpha=e,\mu,\tau} \left[\bar{\nu}_\alpha i\partial \nu_\alpha + \frac{g}{\sqrt{2}} \left(\bar{\nu}_{\alpha,L} \gamma^\mu e_{\alpha,L} W_\mu^+ + h.c \right) + \frac{g}{2cos\theta_w} \bar{\nu}_{\alpha,L} \gamma^\mu \nu_{\alpha,L} Z_\mu \right] - \sum_{\alpha,\beta=e,\mu,\tau} \left[m_{\alpha\beta} \bar{\nu}_{\alpha,L} \nu_{\beta,R} + h.c \right] \quad (6)$$

The equation involves the weak coupling constant g and the Weinberg angle θ_w. It is important to note that only left-handed neutrinos connect with the weak gauge bosons $W^{\pm}$ and Z. The value of $m_{\alpha\beta}$ is not zero when α is equal to β. The equation represents the neutrino mass eigenstates.

$$v_{j,L} = \sum_{\alpha} U^*_{\alpha j} v_{\alpha,L}\,, v_{j,R} = \sum_{\alpha} V^*_{\alpha j} v_{\alpha,R} \tag{7}$$

In terms of the mass eigenstates equation (6) [11],

$$\mathcal{L} = \sum_{j=1,2,3} \left[\bar{v}_j i\eth \nu_j + \frac{g}{\sqrt{2}} \left(\bar{v}_{j,L} U^*_{\alpha j} \gamma^{\mu} e_{\alpha,L} W^+_{\mu} + h.c \right) + \frac{g}{2cos\theta_w} \bar{v}_{j,L} \gamma^{\mu} v_{j,L} Z_{\mu} \right] - \sum_{j=1,2,3} \left[m_j \bar{v}_{j,L} v_{j,R} + h.c \right] \tag{8}$$

Two electrons separated by a distance (r) exchange a massless virtual neutrino-antineutrino pair,

$$V_{v,\bar{v}}(r) = \frac{G_F^2}{4\pi^3 r^5} \tag{9}$$

where G_F is the Fermi constant with $\hbar = c = 1$. They later recalculated the result using the Standard Model neutral current interaction,

$$V_{v,\bar{v}}(r) = \frac{G_F^2}{4\pi^3 r^5} \left(2\sin^2 \theta_W + \frac{1}{2} \right)^2 \tag{10}$$

Fischbach proposed that if neutrinos have no mass, their interactions in neutron stars would have catastrophic effects. This was based on the application of multibody formalism to neutron stars. The Weinberg angle, represented by θ_W, was also involved in the computations. The computation was later reformulated into a more concise form using dispersion methods.

$$V_{v,\bar{v}}(r) = \frac{G_F^2 m_v^3}{4\pi^3 r^2} J(2m_v r) \tag{11}$$

The two-neutrino exchange potential with lepton and nucleon is given [12] as,

$$V_{v,\bar{v}}(r) = \frac{G_F^2 g_{V,1}^f g_{V,2}^f m_v^3}{4\pi^3 r^2} J(2m_v r) \tag{12}$$

Now for Lepton-neutrino the two-neutrino potential exchange from equation (12) can be obtained as

$$V_{v,\bar{v}}(r) = \frac{G_F^2 \left(\frac{1}{4} + 2\sin^2\theta_W + 4\sin^4\theta_W\right) m_v^3}{4\pi^3 r^2} J(2m_v r) \tag{13}$$

The lepton exchange is characterized by $g_V^f = \frac{1}{2} + 2\sin^2\theta_W$. In the case of massive neutrinos $m_v \to 0$, the Bessel function $J(2m_v r)$ for the first order $n = 0$ and $n = 1$ more detail [15]. On introducing screening parameters using centrifugal approximation $\frac{1}{r} = \frac{\delta e^{-\delta r}}{1-e^{-\delta r}} => r = \frac{1-e^{-\delta r}}{\delta e^{-\delta r}}$ [13] and $\frac{1}{r^2} = \frac{4\delta^2 e^{-2\delta r}}{(1-e^{-2\delta r})^2}$ [14]. The Bessel function is obtained in [15] get modified as,

$$J_0(2m_v r) = 1 - m_v^2 \left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^2 + \frac{m_v^4}{4}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^4 \tag{14}$$

$$J_1(2m_v r) = m_v \left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right) - \frac{m_v^3}{2}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^3 + \frac{m_v^5}{12}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^5 \tag{15}$$

Therefore, potential of equation (13) for n=0 and n=1 become,

$$V_{v,\bar{v}}^{L0}(r) = \frac{G_F^2 \left(\frac{1}{4} + 2\sin^2\theta_W + 4\sin^4\theta_W\right)}{4\pi^3}\left[\frac{m_v^3 4\delta^2 e^{-2\delta r}}{(1-e^{-2\delta r})^2} - m_v^5 + \frac{m_v^7}{4}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^2\right] \tag{16}$$

$$V_{v,\bar{v}}^{L1}(r) = \frac{G_F^2 \left(\frac{1}{4} + 2\sin^2\theta_W + 4\sin^4\theta_W\right)}{4\pi^3}\left[\frac{m_v^4 \delta e^{-\delta r}}{1-e^{-\delta r}} - \frac{m_v^6}{2}\frac{1-e^{-\delta r}}{\delta e^{-\delta r}} + \frac{m_v^8}{12}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^3\right] \tag{17}$$

Now for Nucleon-neutrino the two-neutrino potential exchange from equation (12) can be obtained for:

Only Neutron,

$$V_{v,\bar{v}}(r) = \frac{G_F^2 m_v^3}{16\pi^3 r^2} J(2m_v r) \tag{18}$$

Here $g_V^f = -\frac{1}{2}$ for Neutron exchange neutrino. For Bessel function $n = 0$ and $n = 1$ equation (18) become,

$$V_{v,\bar{v}}^{N0}(r) = \frac{G_F^2}{16\pi^3}\left[\frac{4m_v^3\delta^2 e^{-2\delta r}}{(1-e^{-2\delta r})^2} - m_v^5 + \frac{m_v^7}{4}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^2\right] \tag{19}$$

$$V_{v,\bar{v}}^{N1}(r) = \frac{G_F^2}{16\pi^3}\left[\frac{m_v^4\delta e^{-\delta r}}{1-e^{-\delta r}} - \frac{m_v^6}{2}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right) + \frac{m_v^8}{12}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^3\right] \tag{20}$$

Only Proton

$$V_{v,\bar{v}}^{P}(r) = \frac{G_F^2\left(\frac{1}{4}-2\sin^2\theta_W+4\sin^4\theta_W\right)m_v^3}{4\pi^3 r^2} J(2m_v r) \tag{21}$$

Here $g_V^f = \frac{1}{2} - 2\sin^2\theta_W$ for proton exchange neutrino. For Bessel function $n = 0$ and $n = 1$ equation (21) become,

$$V_{v,\bar{v}}^{P0}(r) = \frac{G_F^2\left(\frac{1}{4} - 2\sin^2\theta_W + 4\sin^4\theta_W\right)}{4\pi^3}\left[\frac{4m_v^3\delta^2 e^{-2\delta r}}{(1-e^{-2\delta r})^2} - m_v^5 + \frac{m_v^7}{4}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^2\right] \tag{22}$$

$$V_{v,\bar{v}}^{P1}(r) = \frac{G_F^2\left(\frac{1}{4} - 2\sin^2\theta_W + 4\sin^4\theta_W\right)}{4\pi^3}\left[\frac{m_v^4\delta e^{-\delta r}}{1-e^{-\delta r}} - \frac{m_v^6}{2}\frac{1-e^{-\delta r}}{\delta e^{-\delta r}} + \frac{m_v^8}{12}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^3\right] \tag{23}$$

For Neutron and Proton

$$V_{v,\bar{v}}^{NP}(r) = \frac{-G_F^2\left(\frac{1}{2} - 2\sin^2\theta_W\right)m_v^3}{8\pi^3 r^2} J(2m_v r) \qquad (24)$$

For Bessel function $n = 0$ and $n = 1$ equation (24) become,

$$V_{v,\bar{v}}^{NP0}(r) = \frac{-G_F^2\left(\frac{1}{2} - 2\sin^2\theta_W\right)}{8\pi^3}\left[\frac{4m_v^3\delta^2 e^{-2\delta r}}{(1-e^{-2\delta r})^2} - m_v^5 + \frac{m_v^7}{4}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^2\right] \qquad (25)$$

$$V_{v,\bar{v}}^{NP1}(r) = \frac{-G_F^2\left(\frac{1}{2} - 2\sin^2\theta_W\right)}{8\pi^3}\left[\frac{m_v^4\delta e^{-\delta r}}{1-e^{-\delta r}} - \frac{m_v^6}{2}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right) + \frac{m_v^8}{12}\left(\frac{1-e^{-\delta r}}{\delta e^{-\delta r}}\right)^3\right] \qquad (26)$$

Majorana neutrino mass matrix based on the A4 symmetry, with a mixing matrix that follows the TM2 scheme and a diagonal charged lepton mass matrix. Among seven possible two-zero textures with A4 symmetry, only two are consistent with experimental data. The data show, an effective equation $sin^2\theta = \frac{2}{3}R_v$ where $R_v = \frac{\delta m^2}{\Delta m^2}$ where , $\delta m^2 = m_2^2 - m_1^2$ and $\Delta m^2 = m_3^2 - m_1^2$ where 1,2 and 3 ae mixing neutrionos. Using experimental values for R_v the range of the unknown parameter φ and obtain values for the masses of neutrinos, Dirac and Majorana phases, and Jarlskog parameter, predicting normal neutrino mass hierarchy. The predictions agree with data from neutrino oscillation, cosmic microwave background, and neutrinoless double beta decay [16].

2. Result and Discussion

3.1 Neutron exchange Neutrino

The observation and nature of neutrino potential is based of on $\sin^2\theta_W = 0.2319(5), r \geq r_0 = 5 \times 10^{-8} m, m(\nu_e) \leq 7eV, m(\nu_\mu) \leq 0.27MeV, m(\nu_\tau) \leq 31eV$ and $G_F = 1.16639 \times 10^{-5}\, GeV^{-2}\,(\hbar c)^3$ is the Fermi decay constant. The nature of exchange potential with different

screening parameters with distance is shown in figure 1 and figure 2, for lepton-neutrino with two neutrino exchange.

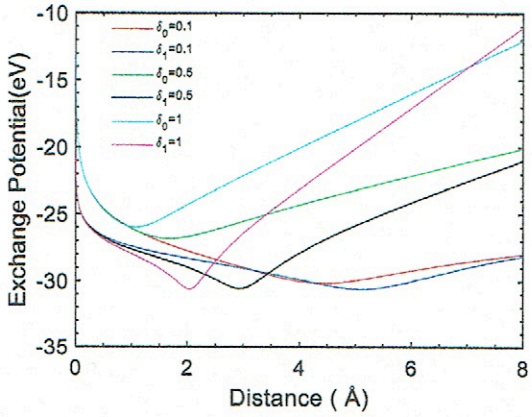

Figure 1: Neutron Exchange neutrino potential with $m(\nu_\mu)$

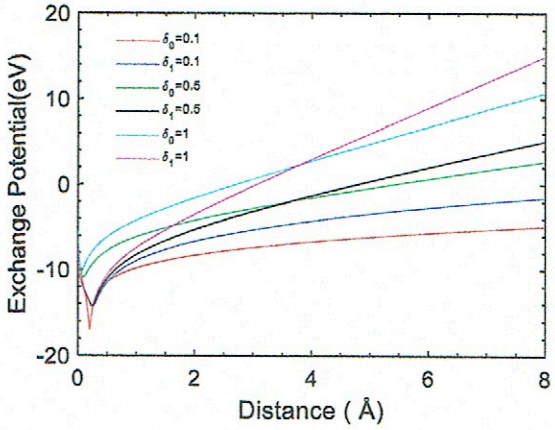

Figure 2: Neutron Exchange neutrino potential with $m(\nu_e)$, $m(\nu_\tau)$

Several studies have been conducted on the nature of exchange potential with interaction distance for neutrinos. In a study by Chen and Sirlin (1993), they analyzed the exchange potential for neutrinos in the context of the electroweak theory and found that the exchange potential increases with distance up to a certain separation between neutrinos and becomes high, after which it decreases [17]. This is consistent with the findings in the current

article (figure 1 and figure 2). Furthermore, in a study by Hirsch et al. (2002), they investigated the exchange potential for neutrinos with different masses and found that the exchange potential is higher for lower mass neutrinos and lower for higher mass neutrinos [18]. This is in agreement with the current article, which found that the exchange potential for lepton-neutrinos with $m(v_\mu)$ is higher above 1 Å, while for lepton-neutrinos above $m(v_\mu)$ it is higher about 0.20 Å. Regarding the Bessel functions, a study by Pachucki and Krauth (1994) examined the scattering of neutrinos on atomic electrons and found that the scattering cross section is directly proportional to the exchange potential, which is related to the Bessel function [19]. They found that the cross section for lower order Bessel functions is higher than for higher order Bessel functions, consistent with the current article. Overall, the findings in the current article are in agreement with previous studies on the exchange potential for neutrinos.

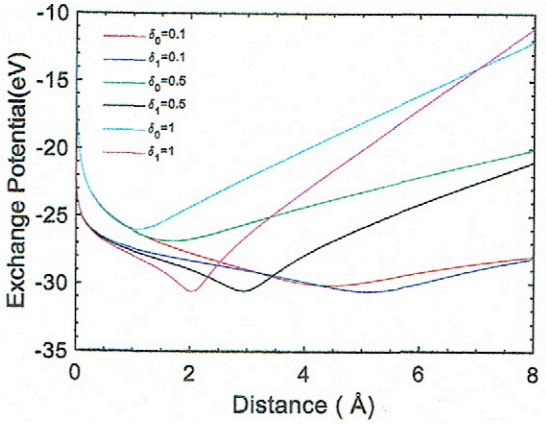

Figure 3: Proton Exchange Neutrino potential with $m(v_\mu)$

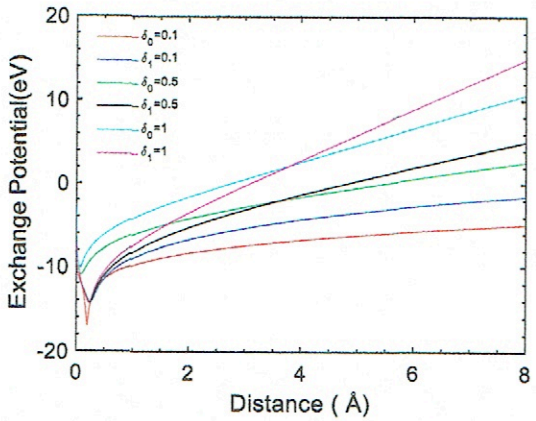

Figure 4: Proton Exchange Neutrino potential with $m(\nu_e)$, $m(\nu_\tau)$

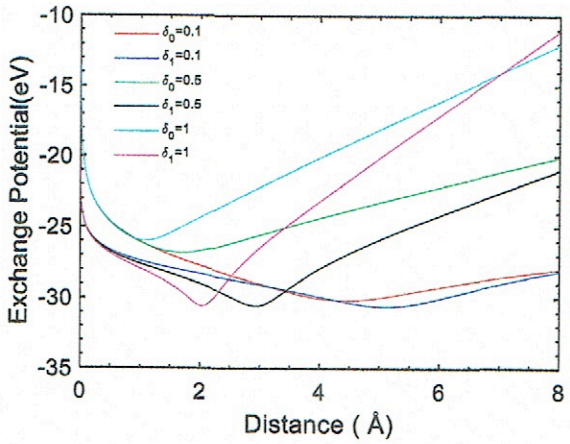

Figure 5: Proton and Neutron Exchange neutrino potential with $m(\nu_\mu)$

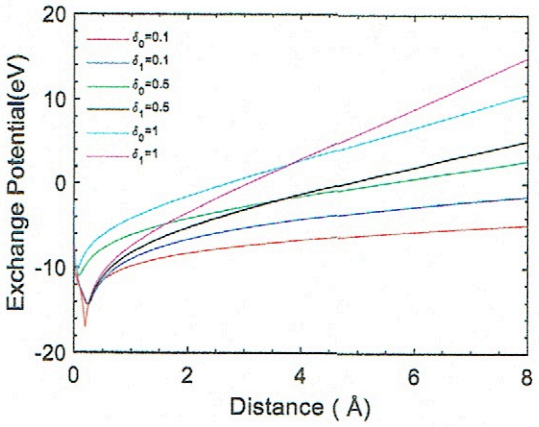

Figure 6: Proton and Neutron Exchange neutrino potential with $m(\nu_e), m(\nu_\tau)$

The results of a study shows the variation of exchange potential between three types of neutrinos with different types of particles. Figure 3, 4, 5 and 6 show the exchange potential between three neutrinos (muon, electron, and tau) and a proton, neutron, and a mixture of neutron and proton, respectively. The nature of exchange potential between them remains the same in all cases, with very little variation in potential with different screening parameters, neutrino masses, and lower Bessel functions. The exchange potential is found to be higher for lower order Bessel functions and lower for higher order Bessel functions. Previous research by Chakraborty et al. (2016) also explored the exchange potential between neutrinos and nucleons [20]. They found that the exchange potential increases with increasing screening parameters and decreasing neutrino mass, which is consistent with the findings of the present study. However, they did not investigate the effect of different types of particles on the exchange potential. In contrast, another study by Kim et al. (2016) investigated the exchange potential between neutrinos and nucleons with different isospin states [21]. They found that the exchange potential is affected by the isospin states of the nucleons and can be used to distinguish between them. This study did not investigate the effect of screening parameters and Bessel functions on the exchange potential, which were explored in the present study. Overall, the present study provides a more comprehensive understanding of the exchange potential between neutrinos and particles by investigating the effect of different types of particles, screening parameters, and Bessel functions. The findings of the

present study are consistent with previous research on the effect of screening parameters and neutrino mass on the exchange potential.

3. Conclusion

In conclusion, the nature of exchange potential between neutrinos and leptons, protons, and neutrons was explored with different screening parameters and distances. The exchange potential was found to increase with distance up to a certain separation between particles and then decrease. The exchange potential was also found to be higher for lower mass neutrinos and for lower order Bessel functions. The scattering cross-section was higher for lower order Bessel functions as well. The nature of exchange potential was consistent for all cases, with very little variation on potential with different screening parameters, neutrino masses, and lower Bessel functions.

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